Assignment 2

1. Given the numbers 10, 13, 8, 5 and 8, what is the average of these five numbers? Given the weights 0.1, 0.4, 0.2, 0.2 and 0.1, what is the weighted average of these numbers.

2. We discussed that a convex combination (a weighted average where all weights are between zero and one and sum up to one) must have the property that the weighted average must be some value between the minimum and maximum values being averaged. Suppose you got 40 on the mid-term examination and 60 on the final examination of this course. Find weights so that, with these grades, you a) get zero in this course, and b) get 100 in this course.

3. Suppose you had a sensor and you would like to record the average of the last n readings. One approach would be to have n memory locations that store the last n readings, and then calculate the sum of those n readings and dividing the result by n. If n was large, this would require significant effort with each step, and even if n is reasonably small (say, $n = 10$), it still would require processing power. Devise a scheme so that the average can be calculated with only a fixed number of arithmetic operations with each new reading, instead of n operations.

4. We saw how we can approximate the square root of two by iterating $x/2 + 1/x$. Demonstrate that this is true by showing that the square root of two is a solution to $x/2 + 1/x = x$.

5. Given the value n, what does iterating $x/2 + n/(2x)$ appear to converge to? Prove this by showing that the solution you propose is indeed a solution to $x/2 + n/(2x) = x$.

6. Suppose we wanted to solve $x^3 - x - 1 = 0$. Find some rewriting of this equation so that fixed-point iteration converges to the real root of this equation.

7. Suppose we want to solve $x^3 - x^2 - x - 1 = 0$. Find some rewriting of this equation so that fixed-point iteration converges to the real root of this equation.

8. Apply Gaussian elimination with partial pivoting and then apply backward substitution to find the solution to

$$\left(\begin{array}{cc:c} -2.4 & -5.6 & 4.0 \\ 3.0 & 2.0 & 5.0 \end{array}\right).$$

9. Apply Gaussian elimination with partial pivoting and then apply backward substitution to find the solution to

$$\left(\begin{array}{ccc:c} -1.2 & 4.4 & 4.9 & -0.2 \\ 2.4 & 0.2 & 0.4 & 2.8 \\ 4.0 & 2.0 & -3.0 & -6.0 \end{array}\right)$$

10. Apply Gaussian elimination with partial pivoting and then apply backward substitution to find the solution to

$$\left(\begin{array}{cccc:c} 5.0 & 2.0 & 0 & 0 & 1.0 \\ 4.5 & 7.8 & -3.0 & 0 & -17.1 \\ 0 & -4.2 & 3.3 & 5.6 & 9.4 \\ 0 & 0 & 4.0 & 2.0 & 6.0 \end{array}\right)$$

11. Apply two steps of Jacobi's method to find an approximation of the solution to:

$$\begin{pmatrix} 10 & 2 \\ 2 & 10 \end{pmatrix}\mathbf{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

12. Apply one step of Jacobi's method to find an approximation of the solution to:

$$\begin{pmatrix} 5 & 2 & 1 \\ 2 & 10 & -3 \\ 1 & -3 & 20 \end{pmatrix}\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

13. The following system is given with the solution:

$$\begin{pmatrix} 2 & 5 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}.$$

If you were to try to apply Jacobi's method to find the solution, does it seem to converge? Why does this happen?

14. Given the matrix $A = \begin{pmatrix} 10 & 9 \\ 9 & 8 \end{pmatrix}$ has the solution to $A\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ as $\mathbf{u}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. What is the solution to the problem $A\mathbf{u}_2 = \begin{pmatrix} 0.9 \\ 1.1 \end{pmatrix}$? What is $\left\| \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0.9 \\ 1.1 \end{pmatrix} \right\|_2$? What is $\|\mathbf{u}_1 - \mathbf{u}_2\|_2$. Recall that $\| \; \|_2$ is the 2-norm or Euclidean norm where $\|\mathbf{u}\|_2 = \sqrt{u_1^2 + u_2^2}$. Use MATLAB to find the condition number of this matrix.